Validity of double scaling analysis in semi-inclusive processes – J/ψ production at HERA

Tapobrata Sarkar, Rahul Basu

The Institute of Mathematical Sciences, Chennai (Madras) 600 113, India (e-mail: sarkar,rahul@imsc.ernet.in)

Abstract. In this paper we check the validity of the ideas of double scaling as given by Ball and Forte in a semi inclusive process like J/ψ production at HERA, in different kinematical regions, for low values of the Bjorken variable x. In particular, we study J/ψ production in the inelastic and diffractive (elastic) regimes using the double scaling form of the gluon distribution functions. We compare these predictions with data (wherever available) and with other standard parameterisations. We find that double scaling holds in the inelastic regime over a larger kinematic region than that given by the analysis of the proton structure function F_2^p. However, in the diffractive region, double scaling seems to suggest an admixture of hard pomeron boundary conditions for the gluon distribution, while predicting a steeper rise in the cross section than suggested by present data.

1 Introduction

The systematic study of the dynamics of perturbative QCD at low values of the Bjorken variable x has thrown up many new ways of analysing the structure of the proton through a measurement of the structure function F_2^p. In particular, one of the methods that provides a clean study of the scaling properties of structure functions as Q^2 (the virtuality of the photon) increases and x decreases is the so-called double scaling analysis of Ball and Forte [1]. In their work, in the kinematic region of very large values of the photon-proton centre of mass energy squared, S, the Altarelli-Parisi (DGLAP) evolution equation were solved and it was shown that the gluon distribution (which is the dominant partonic distribution in the region of interest) shows double scaling in terms of the variables

$$\sigma \equiv \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}} \quad \rho \equiv \sqrt{\frac{\ln x_0/x}{\ln t/t_0}} \qquad (1)$$

where the starting scale for Q_0^2 in $t_0 \equiv \ln \frac{Q_0^2}{\Lambda^2}$ can be just a little more than $Q_0^2 = 1 GeV^2$, $\Lambda = \Lambda_{QCD}$, $x_0 = 0.1$, typically. This asymptotic analysis is a straightforward consequence of perturbative QCD evolution of structure functions at low x, and provides excellent agreement with the data for $Q^2 \geq 5 GeV^2$. However, taking into account the fact that other analyses, the most common being the BFKL approach [2], also provide a mechanism for predicting the behaviour of F_2 at low x, a case has been made for studying the validity of some of these different approaches in processes that are not as inclusive as F_2. One such process where the validity of the double scaling hypothesis has

not been tested is J/Ψ production. In this paper, we propose to remedy this by studying J/Ψ production at high centre of mass energies of the kind available at HERA, using the double scaling form for the gluon distribution function.

The issue of J/Ψ production at large centre of mass energies available at HERA ($\sqrt{S} \sim 200 GeV$) has become an important aspect of QCD because, generally, production of heavy quarkonium states in high-energy collisions is an important tool for the study of its various perturbative and non-perturbative aspects. The production of heavy quarks in a hard scattering process can be calculated perturbatively. However the subsequent creation of a physical bound state (J/Ψ or other quarkonia) involves non-perturbative QCD. This latter process is treated in a factorised approach where the short and long distance physics are separated out. In particular, the evolution of a quark-antiquark pair into a physical quarkonium state is described by the color singlet or color octet models to which we will return in more detail later.

In this paper, we shall address the issue of J/Ψ production at low values of the Bjorken variable x. At low values of x, the gluon distribution plays a dominant role in the determination of the cross section as mentioned earlier. In particular, we concentrate on the specific kinematic region of very large values of the photon-proton center of mass energies, $\sqrt{S}$, in e-p collisions. We calculate the J/Ψ production cross sections at low x, using the asymptotic form for the gluon distribution function, given by the (leading order) double scaling analysis in different kinematical regimes, namely for inelastic and diffractive scattering. Till now, most calculations for J/Ψ production use the standard parametrisations for the gluon distribu-

tions, based on a fit to the known data. The advantage of using the double scaling form is that it is, in a certain sense, derived from QCD and has no fitting parameters other than the starting values of Q^2 and x from where the distributions are evolved. We have, in fact, checked the dependence of our predictions on these starting values in order to be able to quantify the degree of uncertainty that arises from these parameters. for diffractive J/Ψ photoproduction.

The paper is organized as follows. In Sect. 2, we briefly summarize the phenomenon of double asymptotic scaling of the gluon distribution function. In Sect. 3, we present our results for J/Ψ production in the inelastic regime. We then, in the next section move on to the elastic regime, where BFKL dynamics is supposed to be dominant and make our predictions regarding the cross sections. Although it may seem inconsistent to use the gluon distribution function obtained by solving the Altarelli-Parisi evolution equation in the double scaling limit in the diffractive regime where BFKL dynamics takes over, it may be pointed out that these distribution functions are merely used as parametrizations of the data.

In the last section we discuss our results and various open problems.

All the experimental data included in this paper have been obtained from [3] except where otherwise stated.

2 Gluon distribution at low x

There have been two distinct approaches to the study of the HERA data, the 'standard' Altarelli Parisi (DGLAP) [4] evolution equation approach and the attempt to reconcile the data with BFKL dynamics [2]. In order to study the effects of summing $\alpha_s \ln \frac{1}{x}$ unaccompanied by $\ln Q^2$, the preferred approach has been to study the BFKL equation which generates a singular $x^{-\lambda}$ behaviour for the unintegrated gluon distribution $f(x, k_T^2)$, with $\lambda = \alpha_s 4 \ln 2$ (for fixed, not running α_s). For running α_s, $\lambda \simeq 0.5$. [2]. From this an asymptotic form for $F_i (i = 2, L)$ can be inferred. We shall come back to the issue of BFKL dynamics in more detail when we deal with diffractive J/Ψ photoproduction, later in the paper. In the other approach one attempts to describe the data through the DGLAP evolution equation to the next-to-leading order approximation. Here again, the data imply a steep gluon distribution with the gluon density rising sharply as x decreases, even for comparatively low values of Q^2. Ball and Forte [1] have used this approach to exhibit the scaling properties of the gluon distribution function and hence the proton structure function F_2 at low x, generated by QCD effects and have shown that the HERA measurement of $F_2(x, Q^2)$ is well explained by this approach. In [1] it was shown that the solution of the traditional DGLAP equations in the low x regime exhibits double scaling in terms of the variables defined in (1). Double asymptotic scaling results from the use of the operator product expansion and the renormalisation group at leading (and next-to-leading) order, and predicts the rise of F_2 on the basis of purely perturbative QCD evolution. The asymptotic behaviour of the gluon

density in this approach is given (to leading order) by

$$xg(\sigma, \rho) \sim \frac{N}{\sqrt{4\pi\gamma\sigma}} exp \left[2\gamma\sigma - \delta(\frac{\sigma}{\rho}) \right] \left(1 + O(\frac{1}{\sigma}) \right) \quad (2)$$

which leads to the asymptotic behaviour of F_2:

$$F_2^p(\sigma, \rho) \sim N\frac{\gamma}{\rho} \frac{1}{\sqrt{4\pi\gamma\rho}} exp[2\gamma\rho - \delta(\frac{\sigma}{\rho})]$$
$$\times [1 + O(\frac{1}{\sigma})] \quad (3)$$

where

$$\gamma \equiv 2\sqrt{\frac{n_c}{\beta_0}} \quad \beta_0 = 11 - \frac{2}{3}n_f$$

and

$$\delta \equiv (1 + \frac{2n_f}{11n_c^3})/(1 - \frac{2n_f}{11n_c})$$

n_c and n_f being the number of colors and the number of flavors respectively. N is a constant that depends on the input gluon distribution. In our analysis, we have taken $N = 3.24$, corresponding to soft boundary conditions on the gluon [1] as favoured by HERA data [5], in the region of x that we are interested in, for inelastic J/Ψ production. For J/Ψ photoproduction in the diffractive scattering regime, we shall see that harder boundary conditions are preferred and in this kinematical region, we have taken an intermediate pomeron, corresponding to $N = 1.45$.

In our calculation, for the J/Ψ production cross sections in this double scaling limit, at small values of x, we have used the values $Q_0^2 = 1.12 GeV^2$, $\Lambda = 0.248 GeV$ and the starting value of $x_0 = 0.1$. For the values of the variables x and Q^2 for which $\sigma^2 > 1$ and $\rho^2 > 2$, double scaling of the proton structure function F_2 is confirmed to a fair degree. However, in the semi-inclusive J/Ψ production process, we shall not restrict ourselves to these kinematical regimes, but rather try to check the region of validity for double scaling where data is available.

3 The inelastic region

In the color singlet model approach, the dominant contribution to the quarkonium production comes from quark-antiquark or gluon-gluon fusion. The fusion process produces a heavy quark pair which forms a physical color singlet state with an appropriate color singlet projection. This fusion contribution to J/Ψ production in the color singlet model has been computed in [6]. It takes place via the subprocess $\gamma g \rightarrow J/\Psi g$ where the gluon in the final state of the partonic scattering process carries off the color charge. The final $c\bar{c}$ state is a color singlet and the cross section for the process $\gamma N \rightarrow J/\Psi X$ can be calculated from the above subprocess and is given by

$$\frac{d^2\sigma}{dtdx} = \frac{8\pi\alpha_s^2 \Gamma_{ee}^J m_J^3}{3\alpha} g(x)f(s, t) \quad (4)$$
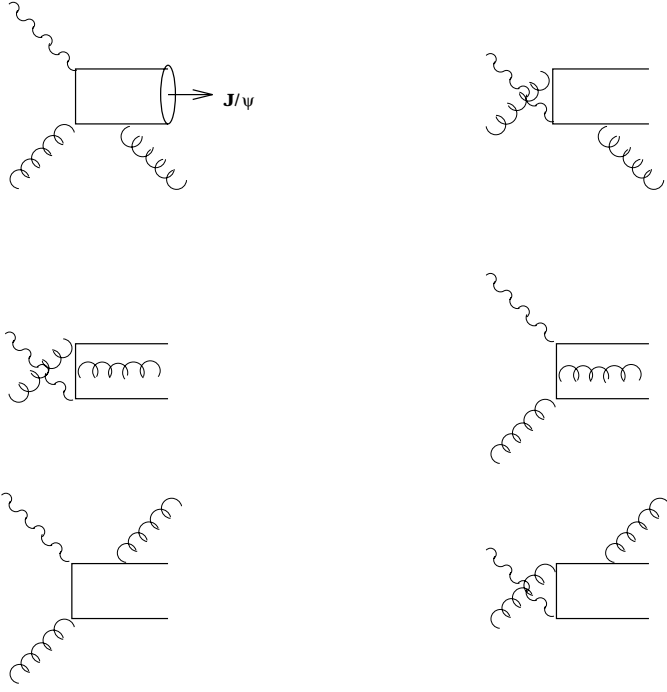
Fig. 1. Color Singlet contribution to inelastic J/ψ production

where,

$$f(s,t) = \frac{1}{s^2}$$
$$\times \left[\frac{s^2(s-m_J^2)^2 + t^2(t-m_J^2)^2 + u^2(u-m_J^2)^2}{(s-m_J^2)^2(t-m_J^2)^2(u-m_J^2)^2} \right]$$

with $s + t + u = m_J^2$, and Γ_{ee}^J the electronic width of the J/ψ which is related to its orbital wave function at the origin ($\Gamma_{ee}^J = 5.26 KeV$ in our calculations). Here, s, t, u are the usual Mandelstam variables for the partonic subprocess and s is related to the total centre of mass energy S by $s = xS$, x being the fraction of the incident nucleon momentum carried by the gluon. It is more convenient to re-express this differential cross section in terms of the variables z the elasticity parameter, and P_T^2, the transverse J/Ψ momentum squared. The elasticity parameter z is defined by

$$z = \frac{P_{J/\psi}.P_N}{Q.P_N}$$

where $Q = P_\gamma$ = momentum of the incident virtual photon. z denotes the fraction of the energy transferred to the J/Ψ in the process and in particular in fixed target experiments, reduces to $z = E_{J/\psi}/E_\gamma$

In terms of z and P_T, we may write the Bjorken variable as

$$x = \frac{1}{S} \left[\frac{m_J^2}{z} + \frac{P_T^2}{z(1-z)} \right] \quad (5)$$

It is more transparent to reexpress this differential cross section in terms of the variables z and P_T^2. In terms of these variables, the differential cross section is given by,

$$\frac{d^2\sigma}{dP_T^2 dz} = \frac{xg(x)z(1-z)m_J^4 A}{[m_J^2(1-z)+P_T^2]^2} \bar{f}(z, P_T^2) \quad (6)$$

where, $A = \frac{8\pi\alpha_s^2 \Gamma_{ee}^J}{3\alpha m_J}$ and the function $\bar{f}(z, P_T^2)$ is given by

$$\bar{f}(z, P_T^2) = \frac{1}{(m_J^2 + P_T^2)^2} + \frac{(1-z^4)}{(P_T^2 + m_J^2(1-z)^2)^2}$$
$$+ \frac{z^4 P_T^4}{(m_J^2 + P_T^2)^2(P_T^2 + m_J^2(1-z)^2)^2} \quad (7)$$

This equation can be used to evaluate the differential cross sections $\frac{d\sigma}{dP_T^2}$, $\frac{d\sigma}{dz}$ and also the P_T^2 integrated total cross section. It may be remarked here that the above expressions show that the cross sections are finite for non zero P_T^2 for all values of z, and for $P_T^2 = 0$, it diverges in the limit when $z = 1$. However, this value of z is excluded from the domain of our interest, as there is a strict upper bound on z for this inelastic process, *viz.* the restriction that the momentum transferred in the partonic subprocess be above the QCD scale so that perturbative QCD is valid; this restriction, $|t| \geq Q_0^2$ gives

$$(1-z) > \frac{Q_0^2 - P_T^2}{Q_0^2 + m_J^2} \quad (8)$$

and hence, corresponding to $Q_0^2 \simeq 1.12 GeV^2$, we have $z \leq 0.90$.

Before we present the results of our calculation, a word about the applicable region of phase space for the double scaling form of the gluon distribution function is in order. We have already remarked that the predictions of the proton structure function F_2^p at HERA shows that asymptotic behaviour sets in, in terms of the variables σ and ρ defined in the beginning, for $\sigma^2 > 1$ and $\rho^2 > 2$. As can be seen from the expression for x, imposing the cuts $z \leq 0.9$, we cannot really go to sufficiently small values of x for large values of P_T^2, which is essential for the validity of the asymptotic form of the gluon distribution. To be well within the double scaling region in x and Q^2, for such large values of P_T^2, one thus has to impose much stronger cuts on z. However, we shall not use such cuts in our analysis as we try to go beyond the limits for σ and ρ as obtained from the analysis of F_2^p.

We have analysed the behaviour of the total cross section and the P_T and z differential cross sections for J/Ψ production. The results of the analysis are shown in Figs. 2–4. In our calculation, we have taken $m_c = 1.4$ and the starting scale x_0 is taken to be 0.1. For the calculation of the total cross section, we have imposed the cuts $0.1 \leq z \leq 0.8$ and $P_T^2 \geq 1 GeV^2$, whereas for the P_T differential cross section, the cut on the elasticity paramter has been imposed as $0.1 \leq z \leq 0.9$ in order to match with experimental data from [3]. Cuts on P_T are generally imposed as in the region of small P_T, the non leading order calculations are not fully under control. Also, the lower bound on z is required in order to suppress contribution to J/Ψ production arising out of B decays. We have checked that on inclusion of a K factor of 1.7 in the total cross section data (as in [11]), the results are in fair agreement with extrapolated data points [3]. From the results it is also clear the one can extend the double scaling validity region to kinematic ranges that go beyond that
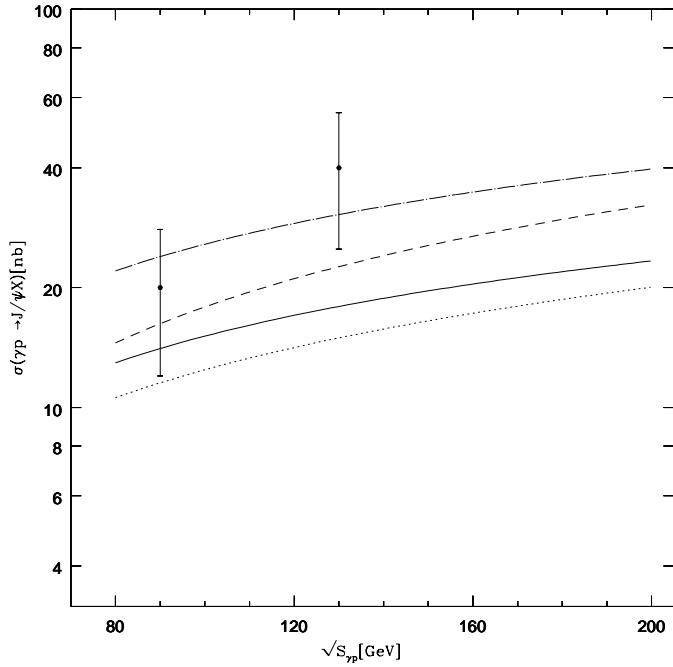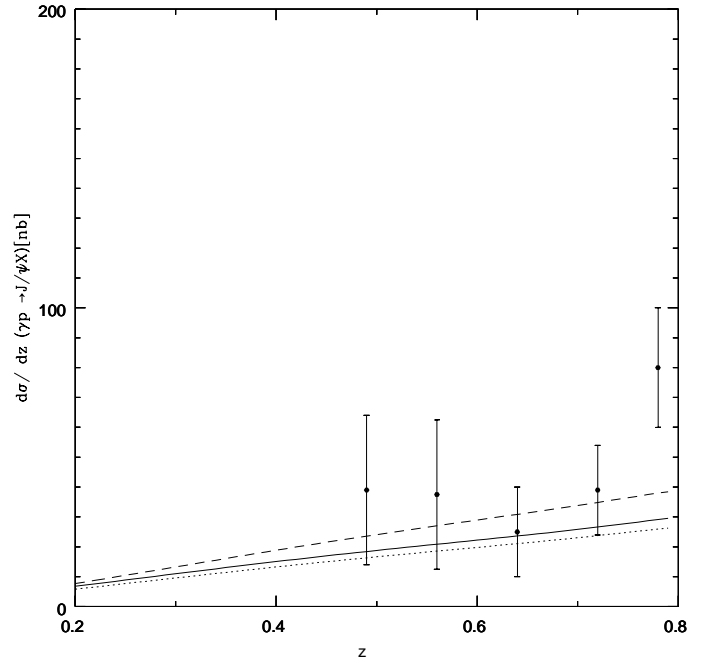
Fig. 2. Total cross section for inelastic J/Ψ production. The *solid line* corresponds to the Ball and Forte gluon distribution, the *dashed line* to the GRV [8] and the *dotted line* to the CTEQ [10] parametrisations. The *dashed-dotted line* shows a fit to the data with the Ball and Forte distribution assuming a K-factor of 1.7
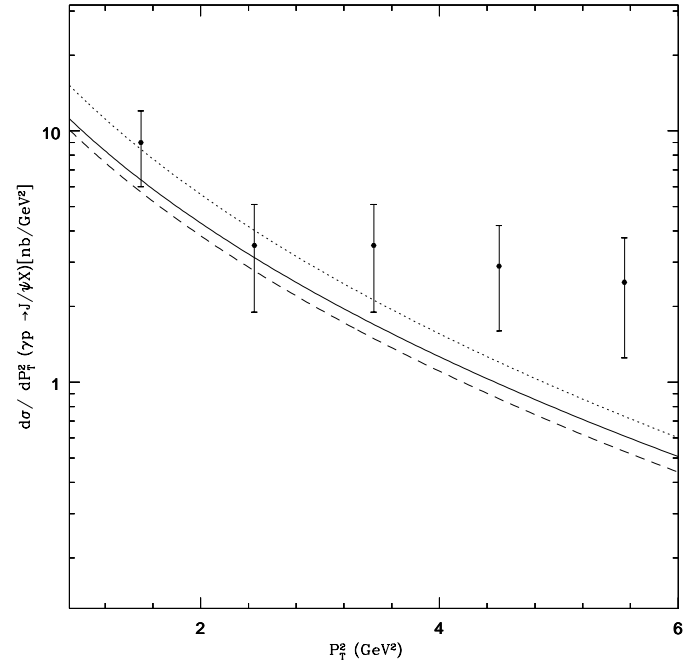

Fig. 3. z Differential cross section for inelastic J/Ψ production at $\sqrt{S_{\gamma p}} = 100 GeV$ The *solid line* corresponds to the asymptotic gluon distribution, the *dashed line* to the GRV fit and the *dotted line* to the CTEQ parametrisation

obtained from the analysis of the proton structure function F_2. Indeed, one gets fairly good agreement with the data for values of the variable $\sigma^2 \geq 0.4$, ρ^2 being typically greater than 2.

We should mention here that in the expression for the total cross section and $d\sigma/dz$, one has to in principle integrate over all values of $P_T^2 \geq 1 GeV^2$. From the expression for x, it is clear that to remain at low values of x, one cannot really go to large values of P_T ($P_T \geq 6 GeV$, say), without severely constraining the inelasticity parameter z, as we had pointed out. However, as can be seen from the expression for the cross section, in the approximation that P_T^2 is large compared to m_J^2, the denominator falls off as P_T^8, and hence the contributions from the large P_T region is negligible for such large values of P_T. Hence, in our numerical calculations, we have restricted ourselves to $P_T \leq 6 GeV$.

The z and P_T^2 distributions are shown in Figs. 3 and 4. From the P_T^2 distribution, it is seen that the double scaled form of the gluon distribution function underestimates the data to some extent, however, the K factor (enhancement factor for the cross sections on inclusion of the NLO corrections) might take this into account at least in part. In the calculation of the differential cross sections also, we have, as mentioned earlier restricted ourselves to the region $P_T^2 \geq 1 GeV^2$ as in the limit of extremely small transverse momentum, there are large negative contributions coming from NLO corrections and the cross section falls sharply as $P_T \to 0$.

Fig. 4. P_T^2 Differential cross section for inelastic J/Ψ production at $\sqrt{S_{\gamma p}} = 100 GeV$. The *solid line* corresponds to the Ball Forte distribution, the *dotted line* to the GRV and the *dashed line* to the CTEQ parametrisations

In our calculations, we have not considered "color octet" contributions to the J/Ψ cross sections. In the color singlet model, that we are considering, one conventionally assumes that the color singlet $c\bar{c}$ state is produced in the $^3S_1$ state initially and then evolves into the physical J/Ψ state. This model however grossly underestimates the J/Ψ production data at the Tevatron [12]. To reconcile with the data, a new mechanism for J/Ψ production was suggested in [13], the color octet mechanism. Whereas in essence the "color singlet" model ignores the relative velocity v between the quarks in the bound state, this quantity is not often negligible and may give rise to considerable amount of corrections to the results. In [14] an expansion of the quarkonium wave function in powers of v and corrections involving the octet components of the wave function have been carried out. However, as has been pointed out in [15] color octet contributions may become important only in the diffractive limit, i.e z close to 1, for small P_T. It was further demonstrated in [15] that at HERA experiments, in the inelastic region, i.e for $z \leq 0.9, P_T^2 \geq 1$, the data can be well accounted for by inclusion of the next to leading order corrections to the cross sections, and hence the color octet contributions in these energy ranges do not seem to play a very significant role. Comparison of the color singlet and color octet contributions to $\frac{d\sigma}{dz}$ with experimental data has been carried out in [15] and this analysis shows that the J/Ψ spectrum is well accounted for by the color singlet model alone and the octet model shows a marked increase in cross section at large values of z, which is clearly in conflict with experimental data. Hence we are justified in dealing only with the color singlet model in our calculations.

The choice of scale as $Q = P_T/2$ would have in part accounted for the K factor mentioned earlier; however, with this choice of scale, small values of P_T in the double scaling regime cannot be reached, as from the form of the gluon distribution function (2) defined in terms of the variables σ and ρ in (1), it is clear that it diverges for $Q = Q_0$. We have thus chosen our scale as the factorization scale, namely $Q^2 = 2m_c^2$.

Also, at HERA energies, contributions to the J/Ψ production cross section might arise from B decays, however these start contributing significantly at extremely low values of z, namely $z \leq 0.1$ [16] We have ignored such small values of z in our analysis, hence this effect will have negligible effect on our results.

Before ending this section, we would like to point out that although we have considered here only the fusion process contribution to J/Ψ production, at large values of P_T, the fragmentation process, i.e the fragmentation of gluons and charm quarks, also become an important source of J/Ψ production. Even though these contributions appear at higher orders of the strong coupling as compared to the fusion contributions, they become important at large values of the J/Ψ transverse momenta due to an enhancement factor of P_T^2/m_c^2, m_c being the charm quark mass. Consequently, fragmentation process contributions may dominate over fusion at comparatively large values of P_T. Fragmentation functions are computed perturba-
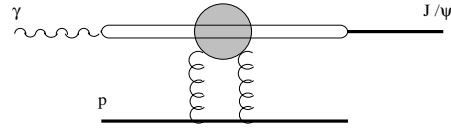

Fig. 5. Diffractive J/Ψ photoproduction

tively, at the initial scale of the order of the charm quark mass and the Altarelli-Parisi equations are then used to resum large logarithms in P_T/m_c. Gluon and charm fragmentation functions have been calculated [17,18] and it has been shown [7] that at HERA experiments, the charm quark fragmentation process is expected to dominate over the quark gluon fusion process at sufficiently values of $P_T \geq 7GeV$. Calculation of the fragmentation functions in the double scaling regime remains an open issue and we hope to address this elsewhere.

4 The elastic region

Having looked at J/Ψ production in the inelastic limit, we now move over to the diffractive region, i.e for values of the elasticity parameter z close to 1. The description of the scattering process via the standard photon gluon fusion mechanism breaks down in this limit, as the elasticity parameter takes values close to 1, and the formulae of the previous section are no longer applicable. In this kinematic regime of diffractive scattering, BFKL dynamics takes over, and in particular, one has to sum over the gluon ladder diagrams as shown in Fig. 5. In this elastic region, the scattering amplitude occurs in a factorized form, [19,20] which is essentially due to the fact that in the proton rest frame, the formation time of the J/Ψ is much greater than the time of interaction of the photon with the proton. The photoproduction amplitude with the incident photon almost on shell is described by a two gluon exchange scattering process where the transverse momentum transfer $t \approx 0$ for the diffractive process. The amplitude for the process in (Fig. 5) was evaluated in the leading log approximation in perturbative QCD in [19] and depends on the *square* of the gluon distribution function. Subsequently, this formula has been improved upon in [20] where the transverse momentum of the gluon ladder is included in the analysis, thus incorporating BFKL [2] dynamics. Dependence on the square of the gluon distribution function makes this cross section extremely sensitive on the latter and a precise measurement of the cross section will shed light on the behaviour of the gluon at very small values of x. It may be mentioned that here we are using the DGLAP evolved gluons in the double scaling limit as a mere parametrization of the data, and hence it is not inappropriate to use them in a regime where BFKL dynamics plays an important role. In [19] the amplitude for the Feynman graphs of (Fig. 5) were evaluated and found to be

$$\frac{d\sigma(\gamma p \to J/\Psi + p)}{dt}$$

$$= [F]^2(t) \frac{\Gamma_{ee}^J \alpha_s^2 m_J^3 \pi^3}{192\alpha} \left[\bar{x} g(\bar{x}, \bar{Q}^2) \frac{2\bar{Q}^2 - |Q_T|^2}{(\bar{Q}^2)^2} \right]^2 \quad (9)$$

where α, α_s and Γ_{ee}^J denote the electromagnetic coupling, the strong coupling and the electronic width of the J/Ψ as before and the quantity F denotes the two gluon form factor that is apriori unknown. To the first approximation it can be treated as the electromagnetic form factor, but its nature is strictly determined experimentally. We shall however restrict ourselves to the domain of photoproduction of J/Ψ in the diffractive regime, i.e where the photon is almost on shell and the momentum transferred, t is extremely small, $t \simeq 0$. At such vanishingly small values of t, this form factor can be approximated to unity [19]. Here, Q_T denotes the transverse momentum of the J/Ψ particle, which is typically small, $\simeq 0.3 GeV$. The quantities $\bar{x}$ and $\bar{Q}^2$ is defined as

$$\bar{Q}^2 = \frac{1}{4} \left(|Q^2| + m_J^2 \right); \bar{x} = 4 \frac{\bar{Q}^2}{S} \quad (10)$$

where S is the photon-proton centre of mass energy. As we wish to consider the photoproduction limit, we set the photon virtuality to be extremely small, and in our calculations, we have chosen the scale $\bar{Q}^2 = 2.4 GeV^2$ In this expression, only longitudinally polarised gluons have been taken into account as these are the only ones that contribute in the extremely high c.m energy limit. We note, first of all, that the dependence of the cross section on the barred quantities ensure that perturbative QCD can be applied even in the photoproduction limit, due to the heavy mass of the J/Ψ. Secondly, the expression for x tells us that at sufficiently large values of S, the extremely small limit of x can be probed.

This formula (9), however has been derived in the leading $\ln Q^2$ approximation, without the inclusion of the transverse momenta of the exchanged gluons. The analysis of the above process including the gluon transverse momenta has been systematically carried out in [20]. In their calculation, the cross section is expressed in terms of the unintegrated gluon distribution, $f_{BFKL}(x, k_T^2)$, which satisfies the BFKL equation, and is related to the conventional integrated gluon density by the relation
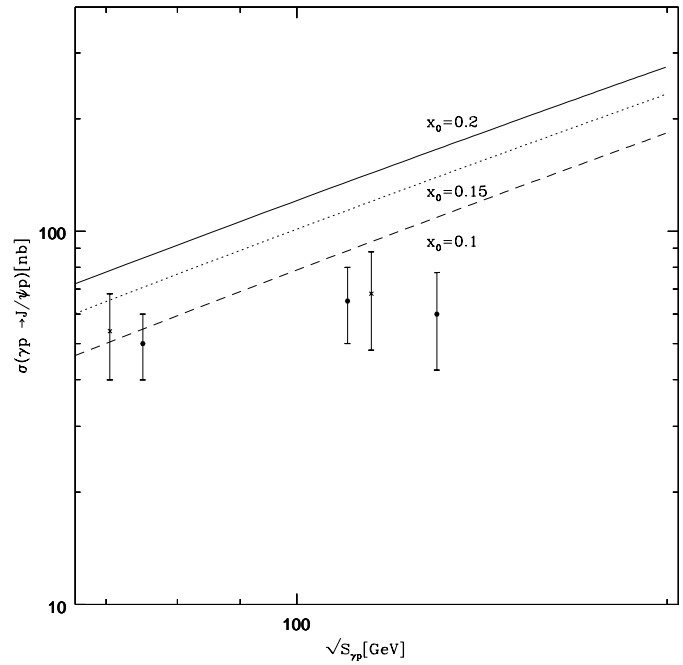
$$xg(x, Q^2) = \int^{Q^2} \frac{dk_T^2}{k_T^2} f_{BFKL}(x, k_T^2) \quad (11)$$

Inclusion of the gluon k_T^2 modifies the scattering cross section to

$$\frac{d\sigma(\gamma p \to J/\Psi + p)}{dt} = \frac{|A|^2}{16\pi} \quad (12)$$

with,

$$A = i 2\pi^2 m_J \alpha_s B \int \frac{dk_T^2}{k_T^2}$$
$$\times \left(\frac{k_T^2}{(\bar{Q}^2)(\bar{Q}^2 + k_T^2)} \right) \frac{dxg(x, k_T^2)}{dk_T^2} \quad (13)$$

Fig. 6. Total cross section for diffractive J/Ψ photoproduction. The *lower*, *middle* and *upper lines* corresponds to $x_0 = 0.1, 0.15$ and 0.2 respectively

and $B^2 = \frac{\Gamma_{ee}^J m_J}{12\alpha}$ The amplitude A can be written in terms of the conventional gluon distribution as

$$A = i 2\pi^2 m_J \alpha_s B \left[\frac{xg(x, Q_L^2)}{\bar{Q}^4} \right.$$
$$\left. + \int_{Q_L^2}^\infty \frac{dk_T^2}{(\bar{Q}^2)(\bar{Q}^2 + k_T^2)} \frac{dxg(x, k_T^2)}{dk_T^2} \right] \quad (14)$$

The lower cutoff Q_L has to be put on the integration due to the undetermined form of the gluon distribution function at vanishingly small values of k_T^2. This form of A can be plugged in (12) in order to obtain the total cross section.

We have obtained the values for the total cross section for diffractive photoproduction of J/Ψ mesons, using (12) and (14). From our analysis, it seems that using the soft pomeron normalisation in this region a la Ball and Forte [1], the double scaling analysis grossly overestimates presently available data, by a factor of about 5. Hence, we conclude that in this region of extremely smally values of x, $x \leq 10^{-3}$, there is some admixture of the hard pomeron boundary condition, and consequently to analyse this region, we have chosen a normalisation corresponding to some intermediate pomeron, namely, $N = 1.45$.

As a lower cutoff for the transverse momenta of the gluon distribution function, we have used the value $Q_L^2 = 1.5 GeV^2$. This value of Q_L^2 has been chosen so that it is just above the starting scale for the gluon distribution (namely $Q_0^2 = 1.12 GeV^2$) in the double scaling analysis. We will come back to the issue of variation of the cross section with this cutoff in a moment. Note that in our calculation, we have considered the forward scatter-

ing amplitude, namely $t = 0$. There may be corrections coming from consideration of a non zero value of the momentum transfer, $|t_{min}|$; however, these corrections are expected to be small in the region of x that we are interested in, namely $x < 0.01$ [20]. One might also consider the corrections arising out of the relativistic effects of the J/Ψ wave function, those coming from treating the quarks in the J/Ψ bound state non relativistically, i.e assuming $m_J = 2m_c$, where m_c is the mass of the charm quark, and also those arising out of radiative corrections to the lowest order process considered here. Analysis of these correction factors have been carried out in [20] and it has been shown that whereas the radiative correction is negligible for soft gluon emission, as is the case here, the total correction factor arising from relativistic effects in the J/Ψ wave function and the motion of the quarks inside the charmonium bound state is of the order of unity.

The results of our analysis is shown in Fig. 6. From the available data points [3], we find that the double scaling form of the gluon distribution function predicts a slightly steeper rise in the elastic cross section than favoured by the experimental data. However, experimental values for larger centre of mass energies than are currently probed may throw light on the scenario for diffractive J/Ψ photoproduction. We have also plotted the variation of the cross section with the choice of the starting scale x_0 in the gluon distribution function in the double scaling limit. It can be seen that change of x_0 causes a considerable amount of fluctuation in the J/Ψ production cross-section. We have checked that there is similar fluctuation in the cross section as one varies the starting scale for the gluon distribution function, namely Q_0^2, though the results are not explicitly indicated here. We have also explored the sensitivity of the cross section to the lower cutoff in the transverse momentum (Q_L) used to compute it. It is found that the cross section is highly sensitive to fluctuations in Q_L^2, varying from about 70% at $\sqrt{S} = 100 GeV$ to nearly 75% at $\sqrt{S} = 200 GeV$. This is in contrast to the smaller variation in cross section noted in [19] using the GRV [8] and MRS [9] gluons, as Q_L^2 is varied.

5 Conclusions

In this paper, we have studied the validity of the double scaling ideas to a specific semi-inclusive process like J/Ψ production. Our results show that for the inelastic region, the data is fairly well accounted for by the double scaling form of the gluon distribution function, even in a region beyond that confirmed by measurement of the proton structure function F_2. However, in the diffractive scattering regime, double asymptotic scaling while predicting a steeper increase in the cross section than from present data indicates the admixture of hard pomeron boundary conditions (as opposed to the soft boundary conditions [1] that we assumed while analysing the inelastic scattering data). As such, it is not very clear, at least from the present analysis as to which behaviour takes over at extremely small values of x, and diffractive J/Ψ photoproduction may pro-

vide some crucial hints, as it is extremely sensitive to the gluon distribution function.

To our knowledge, this is the first attempt to apply the idea of double scaling to a semi-inclusive process. The advantage of using double scaling, as pointed out in the introduction is that it does not need any given form form for the starting gluon distribution $g(x_0, Q_0)$ as with other parametrisations. We have studied whether the region of validity of double scaling in a totally inclusive process like the F_2 measurement changes when a semi-inclusive process like J/Ψ photoproduction is analysed. In the case of diffractive scattering, we have also used the data to constrain the starting value of x, i.e x_0 and found significant change in the value of the total cross section, a similar change being present if one varies the starting scale for the gluon distribution function, i.e Q_0^2.

There have been many suggestions, including a very elegant one by Mueller [21] for studying processes that would clearly distinguish QCD evolution through DGLAP and some other mechanism like BFKL. The basic thrust of all these suggestions has been to study semi-inclusive processes and we have taken a first step in this direction by studying the validity of double scaling in J/Ψ photoproduction where data with reasonably good statistics is already available from HERA. It would be useful to make a systematic study of all such semi-inclusive processes for which data exists, in different resummation schemes (DGLAP, BFKL, CCFM, etc) in order to look for the validity of these different methods. This would then give us a qualitative understanding of the physics of the different processes and consequently that of the various resummation schemes. These are some of the issues for future study.

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